Analysis of Multiuser Diversity in Wireless Data Networks

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Outline

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Wireless Scheduling

Problem: How to dynamically share a wireless link efficiently and fairly amongst various users?

Wireless scheduling paradigm:

- Wireless link is resource limited - bandwidth, spreading codes, power, interference.
- Time-varying.
- Means to achieve capacity often involves varying rates of transmission based upon channel state. Scheduler not aware of radio-conditions cannot wait till good channel state to transmit.

Great benefits to be had with higher layers knowing the physical layer parameters. Thus, joint approach seems more natural - Collins and Cruz'1999, Zhang and Wasserman'2000, Berry and Gallager'2001, Tse'2000, Holtzman'2000, Jalali *et al.*'2000, Chawla *et al.*2000, Shakkottai and Stolyar'2000, Agrawal, *et al.*'2001, Rangsan L and Agrawal'2001.

Efficiency and fairness compete:

1. In 2-G systems only voice was present. Coverage (fairness) was the only concern. Very good conditions for certain users never exploited - nothing to be gained really. Not very efficient!

2. Consider packet data and the policy of service only to the best user(s). Most efficient system. ONLY on-off nature of traffic would allow other users to transmit. No fairness!

It is clear from (1) that such concerns are important only for packetized services.

Time-invariant channels

We consider non-realtime applications - specifically rate-adaptive services - and downlink case.

Let user j get rate $\bar{R}_j$ at time.

Define ρ_j to be the fraction of time (over a long time) that the resources are given to user j.

Throughput of user j - $\rho_j \bar{R}_j$. Choose ρ_j such that

$$\max_{\rho_j} \sum_{j \in \mathcal{J}} U_j(\rho_j \bar{R}_j P_b M_j) \tag{1}$$

subject to:

$$\begin{aligned} \rho_j &\geq 0 \\ \sum_j \rho_j &\leq 1 \end{aligned}$$

$U_j(\cdot)$ is a utility function belonging to the family

$$U^\alpha(x) = \begin{cases} \mathrm{sgn}(\alpha) x^\alpha & \alpha < 1,\ \alpha \neq 0 \\ \log(x) & \alpha = 0 \end{cases}$$

The solution to (1) is

$$\rho_j = \frac{\bar{R}_j^{\beta-1}}{\sum_{i \in \mathcal{J}_b} \bar{R}_i^{\beta-1}}, \forall \alpha < 1,$$

where $\beta = \frac{1}{1-\alpha}$.
Therefore, $\rho_j \propto \bar{R}_j^{\beta-1}$ and throughput $\propto \bar{R}_j^{\beta}$.

Define $C_j = \bar{R}_j^{\beta}$.

Observations:

- $\alpha = 1$, i.e., $\beta = +\infty$ results in $\rho_j = 1$ for $j = \arg\max \bar{R}_j$ or the policy that serves the best user.

- $\alpha = 0$, i.e., $\beta = 1$ results in proportionally fair allocation resulting in ρ_j being equal for all the users - the trade-off assumed in the HDR solution of Tse *et al.*

- $\alpha = -\infty$, i.e., $\beta = 0$ results in users getting a throughput independent of their channels - Equal throughput solution!

Weighted Proportionally Fair Allocation

$$\max_{\rho_j} \sum_{j \in \mathcal{J}} Wt_j \log(\rho_j \bar{R}_j) \qquad (2)$$

subject to the constraints of (1). Solution is

$$\rho_j^* = \frac{Wt_j}{\sum_i Wt_i}.$$

and $Thput_j^* = \frac{Wt_j \bar{R}_j}{\sum_i Wt_i}$.

Solution of (1) same as (2) with weights $Wt_j = \frac{C_j}{\bar{R}_j}$.

Question: How does one achieve the wpf allocation?

Let $W_j(t)$
$=$ amount of data transmitted by user j upto t (present)
$= W_j(t-1) +$ data transmitted at time t.

Let $\bar{W}_j(t) = \frac{W_j(t)}{C_j}$ be the normalized throughput.

Policy: At BS b serve user $j^*(t) = \arg\min_{j \in \mathcal{J}_b} \bar{W}_j(t)$.

We have the following

Proposition 1 *As $t \to \infty$ the algorithm outlined gives the wpf allocation, and hence, the optimal allocation.*

Time-Varying Channels

Assume that user j gets rate $R_j(t)$ at time t.

Try simple approach of using $C_j = \bar{R}_j^{\beta}$.
Then $Thput_j(t) \to Thput_j^*(\bar{R})$ (a.s.) under fairly general conditions (Proof for i.i.d. with $1+\epsilon$ moment).

Is this the best that can be done? NO!

If we transmit to user j when his rate is better than his average, then we can do better.
If there are many users, then it is very likely that some user will be in a good state - **Multiuser Diversity**.

Compute the average effective data rate $\hat{R}_j^{\mathsf{avg}}(t)$ as follows

$$R_j^{\mathsf{avg}}(t+1) = (1-\psi)R_j^{\mathsf{avg}}(t) + \psi R_j(t).$$

Define

$$\begin{aligned} C_j(t) &= w_j[R_j^{\mathsf{avg}}(t)]^{\beta}[\frac{R_j(t)}{R_j^{\mathsf{avg}}(t)}]^{\gamma} \\ &= w_j[R_j^{\mathsf{avg}}(t)]^{\beta-\gamma}[R_j(t)]^{\gamma} \\ &= C_j^1(t)C_j^2(t), \end{aligned} \qquad (3)$$

where $0 \le \gamma \le \beta$.

Let $D_j(t)$ be the amount of data transmitted in frame t for user j. Update $\bar{W}_j$ as follows

$$\bar{W}_j(t+1) = (1-\phi)\bar{W}_j(t) + \phi \frac{D_j(t)}{C_j^1(t)}. \qquad (4)$$

Policy: Rank users in increasing order of $\tilde{W}_j(t) = \bar{W}_j(t)/C_j^2(t)$ and serve user j^* with minimum $\tilde{W}_j(t)$, i.e., $D_j(t) = 1_{[j=j^*]} R_j(t)$.

3 Variations possible:

1. **Variant 1**: Use $C_j^1(t) = C_j(t)$ and $C_j^2(t) = 1$. This resembles the algorithm analysed earlier.

2. **Variant 2**: Use $C_j^1(t) = w_j[\widehat{R}_j^{\text{avg}}(t)]^\beta$ and $C_j^2(t) = [\frac{\widehat{R}_j(t)}{\widehat{R}_j^{\text{avg}}(t)}]^\gamma$.

3. **Variant 3**: Use $C_j^1(t) = 1$ and $C_j^2(t) = C_j(t)$.

The algorithm in Tse'2000 is similar to **Variant 3** with $\beta = \gamma = 1$. We assume that $\psi = \alpha\phi$.

Define

$$Th\tilde{p}ut_j(t+1) = (1-\phi)Th\tilde{p}ut_j(t) + \phi D_j(t). \qquad (5)$$

Performance

CDMA case:

25 cells, 15 users per cell
Data rates - 2400, 1800, 1200, 600, 300 bits per frame (10ms).
Max thput per cell - 1.44 Mbps.

β	Variant 1, $\gamma = 0$	Variant 1	Variant 2	Variant 3
0	294.39, 5.83	294.39, 5.83	294.39, 5.83	294.39, 5.83
1	353.14, 7.97	385.84, 8.85	421.82, 9.27	442.36, 9.77
2	423.43, 10.00	457.74, 10.83	480.86, 10.95	522.15, 11.73

Comparison of average throughput per cell for different values of β and variations of the scheduling algorithm.

In columns 2, 3 and 4: for

- $\beta = 0$, $\gamma = 0$.
- $\beta = 1$, $\gamma = 1$ - HDR proposal.
- $\beta = 2$, $\gamma = 1$.

Stochastic Approximation

[Bucklew, Kurtz, Sethares]:

$$W_{k+1} = W_k + \mu H(W_k, Y_k, U_{k+1}),$$

where W_k represents the parameter estimation errors, Y_k some function of inputs, $U_k = q(W_k, Y_k, \Psi_k)$ is a disturbance process with $\{\Psi_k\}$ i.i.d. and independent of $\{Y_k\}$, and W_0 independent of $\{(Y_k, \Psi_k)\}$.

Define

$$\bar{H}(w, y) = \int H(w, y, u)\eta(w, y, du)$$

where η is the conditional distribution of U_{k+1} given $\mathcal{F}_k$ and

$$\hat{H}(w) = \int \bar{H}(w, y)\nu_Y(dy).$$

If $\{Y_k\}$ statinary and ergodic, $W_\mu(0) \to w_0$ in probability and $\bar{H}(w, y)$ continuous in (w, y), then as $\mu \to 0$, $W_{[t/mu]}$ converges weakly to

$$W(t) = w_0 + \int_0^t \hat{H}(W(s))ds.$$

Variant 1

We have

$$\bar{W}_j(t+1) = \bar{W}_j(t) + \phi\left(1_{[j=j^*]}\frac{R_j^{1-\gamma}(t)}{(R_j^{\text{avg}}(t))^{\beta-\gamma}} - \bar{W}_j(t)\right), \quad (6)$$

where $j^* = \arg\min_j \bar{W}_j(t)$.
As $\phi \to 0$ and for large t, $R_j^{\text{avg}}(t) = \bar{R}_j$ and we get a version of the simple algorithm!!! Thus,

$$Thput_j = \frac{E[R_j]^{\beta-\gamma+1}/E[R_j^{1-\gamma}]}{\sum_i E[R_j]^{\beta-\gamma}/E[R_j^{1-\gamma}]}. \quad (7)$$

We do not exploit multiuser diversity (for small enough ϕ) and for $\gamma = 1$ we get same performance as $\gamma = 0$. In general expect performance to be worse (in terms of sum utility) than $\gamma = 0$ case.

Variants 2 and 3

ODE - V2

$$\frac{d\bar{W}_j}{dt}(t) = E_R[1_{[j=j^*]}\frac{R_j}{(R_j^{\mathsf{avg}}(t))^\beta}] - \bar{W}_j(t) \quad (8)$$

$$\frac{dT\tilde{hput}_j}{dt}(t) = E_R[1_{[j=j^*]}R_j] - T\tilde{hput}_j(t). \quad (9)$$

Scale $\bar{W}_j$ by $E[R_j]^\beta$, then equilibrium solution of V2 and V3 the same!!!

Thus, for small ϕ and large t need to consider only one of them.

Variant 3

Assume $R_j(t)$ is χ-squared distributed with $2n$ degrees of freedom and mean $\frac{1}{\lambda_j}$.

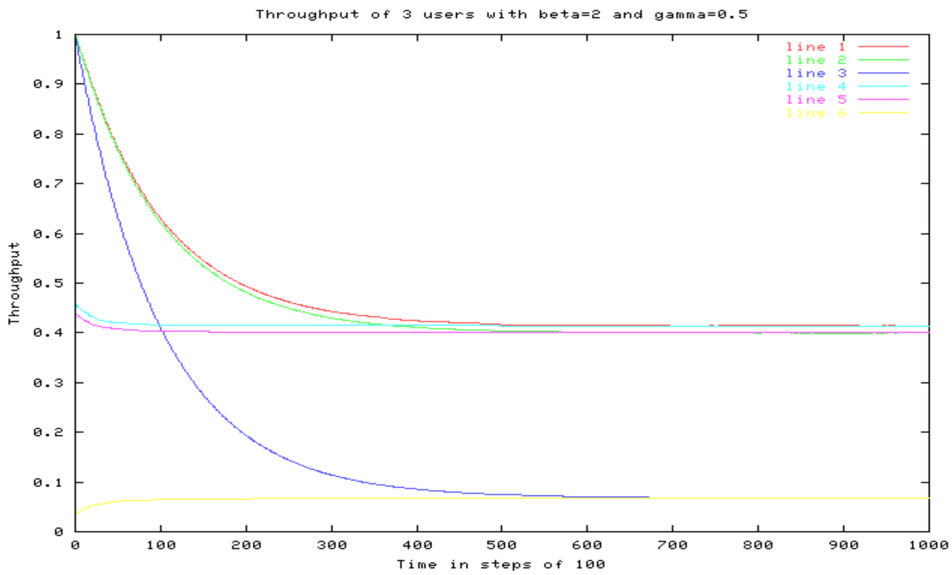
For 2 users equilibrium point given by solution to

$$\begin{aligned}\bar{W}_1 &= \frac{1}{\lambda_1}\left[1-\left(\sum_{l=0}^{n-1}\frac{n+l!}{n!l!}\frac{1}{(1+1/c)^l}\right)\frac{1}{(1+c)^{n+1}}\right],\\ \bar{W}_2 &= \frac{1}{\lambda_2}\left[1-\left(\sum_{l=0}^{n-1}\frac{n+l!}{n!l!}\frac{1}{(1+c)^l}\right)\frac{1}{(1+1/c)^{n+1}}\right],\end{aligned}$$

where

$$c=(\frac{\bar{W}_2}{\bar{W}_1})^{1/\gamma}(\frac{\lambda_2}{\lambda_1})^{\beta/\gamma}.$$

ODE has unique solution, therefore convergence (as $\phi \to 0$) is also in probability. Equilibrium point is locally stable. Numerical investigations indicate that it might be globally asymptotically stable as well.



Variant 1 throughput curves.

Scenario: 3 users with 2-state Markovian rate process.

$$\Phi = \left[\begin{array}{c|c|c} 8.68e-4 & 0.885 & 0.535 \\ 0.165 & 0.984 & 0.39 \end{array} \right]$$

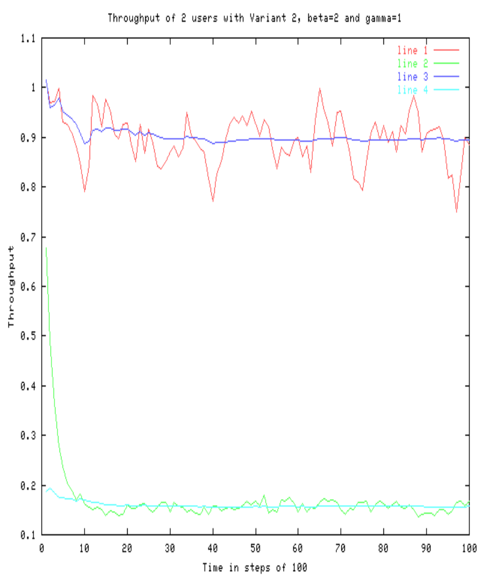
$$R = \left[\begin{array}{c|c|c} 0.994 & 0.972 & 0.235 \\ 0.257 & 0.975 & 0.515 \end{array} \right]$$

$\alpha = 10, \phi = 0.0001, \text{time} = 100000.$

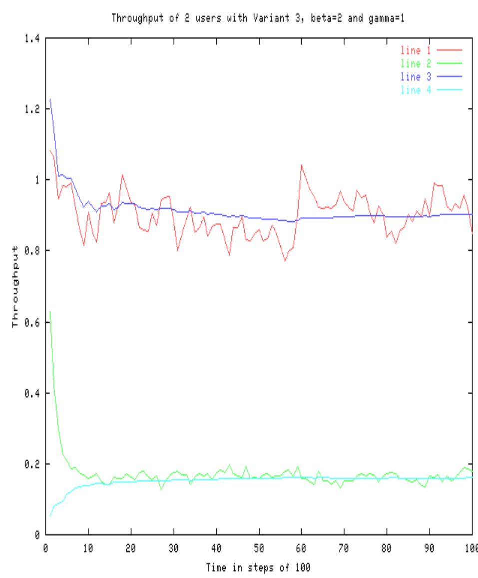
	User 1	User 2	User 3
Theory - $\beta = 2, \gamma = 0.5$	0.414	0.4	0.0678
Sim - $\beta = 2, \gamma = 0.5$	0.414	0.4	0.068
$\beta = 2, \gamma = 0$	0.415	0.401	0.0669

$\bar{W}$: Theory 0.4223, Sim 0.4228, 0.4227, 0.4229.

Scenario: 2 users i.i.d Exponential rates
R=[0.9905 0.294], $\beta = 2$, $\gamma = 1$, ϕ=0.005, time=10000.

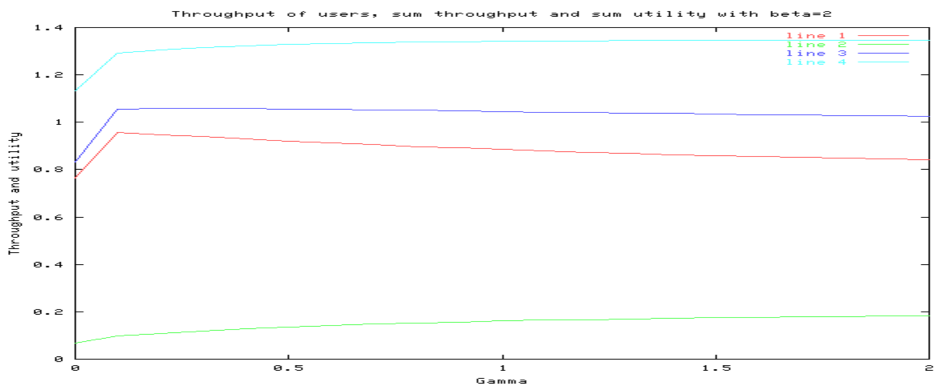


Variant 2 throughput

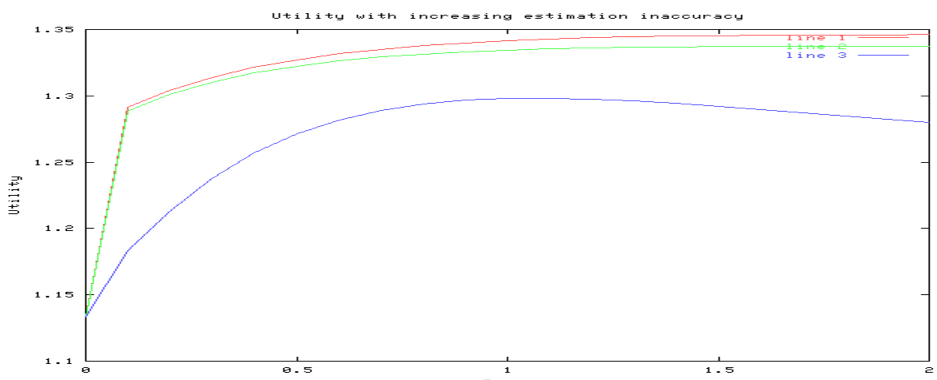


Variant 3 throughput

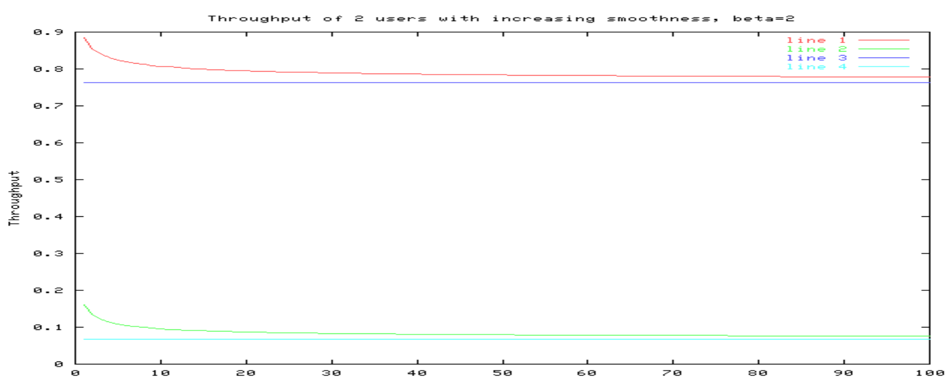
	User 1	User 2
Theory	0.885	0.161
V2 sim	0.895	0.157
V3 sim	0.901	0.161
$\beta = 2, \gamma = 0$	0.764	0.0673



Variant 3 with $\beta = 2$



Variant 3 with $\beta = 2$ and different uncertainity in rates.



Variant 3 with $\beta = 2$ with different distributions.

Future Work

- Approximate solutions to aid in predicting performance or optimizations.

- Investigate the second-order performance - incorporates effect of the correlation structure of channel rate variations.

- Present analysis based on infinite-backlog assumption. What is the capacity region shaped like with queues and arrival processes?

Conclusions

- Significant advantages to using current channel condition information. Tails of the distributions of the rate processes impact the gains.

- For small enough ϕ Variant 1 does not exploit inherent multiuser diversity.

- For small enough ϕ Variants 2 and 3 exhibit the same performance.

- For every $\beta > 0$ there seems to be a best γ the value of which depends on the level of uncertainity of the current rates of the users.