The Role of Search Friction in Networked Markets' Stationarity

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1. INTRODUCTION

The advent of the Internet and the resulting electronic revolution has resulted in incredible speed-ups in trade and commerce. Many time-consuming and arduous transactions have been greatly simplified and sped-up. In modern electronic financial markets, for example, high frequency trading has become a critical component and has resulted in a greatly increased volume of trade. Given the recent financial crisis, studies on the effect of high frequency trading to the market micro-structure and its dynamics has become an active and important research topic.

In this paper, we consider the dynamics of a non-cooperative networked bargaining game as the market trading process and then incorporate search friction into the model. Our goal is to study how search friction influences the market's stability and efficiency in a network economy. Our starting point is the framework introduced in [2], which considers a non-cooperative bargaining process as a micro-market mechanism and incorporates it into a large, competitive networked market. It is shown in [2] that even in the case of the economy becoming large, with more agents at each location so that each agent has a diminishing influence on the economy, the agent's strategy need not converge to a stationary equilibrium resulting in endogenous fluctuations. Another important conclusion drawn from [2] is that a small shock in the economy can be amplified through the network to make a "stable" market change to one that does not have a stable outcome.

In more detail, the model consists of a line network of middleman agents that allow sellers to sell their product to buyers. The network restricts the trades that are allowed only to local interactions, i.e., a middle-man agent can only buy the good from an agent (could be a seller) closer to the seller and can only sell the good to an agent (could be a buyer) closer to the buyer; a seller can only sell the good to an agent that she is connected to and a buyer can only buy the good from an agent that she is connected to. In the set-up analyzed in [2], an agent is chosen at random and based on whether she possesses the good or not, she either sells or buys the good. In either case, the model is such that she can identify all the agents in the next hop with whom she can trade with, if any exist; in other words, her search procedure is *frictionless*: extremely fast and reliable.

In this paper, we introduce search friction into this model of bargaining: there is some likelihood that the chosen agent finds an agent on the other end with whom she cannot trade, either both agents possess the good or both do not. This has the effect of slowing down the trades and results in agents always having a stationary strategy as the economy grows. This holds because an agent who does not have the good needs greater incentives to accept the good if she knows that it will be hard for her to resell the good to a next-in-line agent.

We show that unlike [2], with the presence of search friction stationary equilibrium can be sustained, however, inefficiency persists. In particular, in the new bargaining model, the strategy in which agents trade immediately whenever they meet a potential trading partner cannot be sustained at equilibrium; trade sometimes is delayed.

Our paper continues the line of research in non-cooperative bargaining started by [3]. However, our framework, following [2], differs from most of this literature because we consider the presence of middlemen. The effects of non-cooperative bargaining with middlemen on market outcomes have been studied in [4] and recently in [5]. However, while [4] does not model the bargaining process explicitly and [5] assumes there is a single good in the economy, here we consider a fully dynamic setting.

The abstract is organized as follows. In Section 2 we mathematically describe the model from [2], some of the conclusions therein, and particular form of search friction that we will analyze. The results are presented in Section 3.

2. MODEL

We first describe the framework studied in [2] and then consider a variation of this model that incorporates search friction.

2.1 Local Bargaining Model

Trading Network

The basic underlying structure of the model is a line network, whose nodes are indexed sequentially by $0, 1, \ldots, n+1$. Let $N = \{0, 1, \ldots, n+1\}$. Each node represents a population of $m \geq 1$ agents. (The results extends to the case where each node has a different population.) Node 0 and $n+1$ represent sellers and buyers, respectively, while $\{1, \ldots, n\}$ correspond to intermediaries.

There is one type of indivisible good in this economy. At every period each agent can hold at most one unit of the good (an item). Thus, every time period, an intermediary either has an item or does not have one. In this model, agents trade along the chain. Two consecutive agents j and $j+1$ are *feasible trading partners* if j owns an item and $j+1$ does not own one.

The transaction cost between two consecutive agents i and $i+1$ is $C_i \geq 0$. The value that each buyer gets for an item is $V \geq 0$. We assume trade is beneficial, that is, $V > \sum_i C_i$.

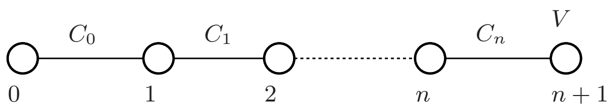

Figure 1: A chain of intermediaries

The Bargaining Process

We consider an infinite horizon, discrete time repeated bargaining game, where agents discount their payoff at rate $0 < \delta < 1$. (The model can be extended for heterogeneous discount rates.) Each period has multiple steps and is described as follows.

Step 1. This is the beginning step of each period. In this step, a node i is picked randomly from the network, then among the m agents, an agent is selected at random.

Step 2. If the selected agent i does not have any feasible trading partner, then the game moves to the next period, which starts from Step 1. Recall that depending on whether i has an item or not, a feasible trading partner of i is either an agent $i+1$ not owning an item or an agent $i-1$ that owns an item.

Step 3. If i has several feasible trading partners, then i picks a random one to bargain with. (The choice of tie-breaking rule can be arbitrary.) To bargain, i suggests a price at which he is willing to trade. If the trading partner refuses, the game moves to the next period. Otherwise, the two agents trade: one agent gives the item to and receives the money from the other. We assume intermediaries are long lived and they do not produce nor consume; they earn money by flipping the good. On the other hand, buyers and sellers exit the game after trade and are replaced by clones.

Step 4. The game moves to the next period, which starts from Step 1.

The game is denoted by $\Gamma(N, m, \vec{C}, V, \delta)$; sometimes we also mention this game as Γ.

Replicated Economy

Given the game $\Gamma(N, m, \vec{C}, V, \delta)$ described as above, the game's replications are defined as a game of the same structure, except the population size is increased by a factor of k, and the time gap between consecutive periods is reduced by a factor of T_k.

DEFINITION 1. *Given the game $\Gamma(N, m, \vec{C}, V, \delta)$ and $k, T_k \in \mathbb{N}^+$, let $m' = k \cdot m$ and $\delta' = \delta^{1/T_k}$. Then the (k, T_k)-replication of Γ, $\Gamma^k_{T_k}(N, m, \vec{C}, V, \delta)$ is defined as $\Gamma(N, m', \vec{C}, V, \delta')$.*

Limit Stationary Equilibrium

To capture the limit behavior of agents, we consider the limit a series of semi-stationary equilibria in finite games. Namely, consider a state of the game to be $\omega \in [0, 1]^n$, that captures the fraction of middlemen at each node holding the good, and consider the set of strategies that depends on an agent's identity, his state and the play of the game: which agent he is bargaining with, who is the proposer and what is proposed. We assume that agents at the same node, in the same state have the same strategy (symmetric), and we allow randomization.

DEFINITION 2. *A pair of state and strategy $(\omega(k), \sigma(k))$ is a* **semi-stationary** *equilibrium in the game $\Gamma^k_{T_k}$, if $\sigma(k)$ is a sub-game perfect equilibrium assuming that agents believe the state of the economy is always $\omega(k)$.*

A pair of state and strategy (ω, σ) is **balanced** *if ω satisfies $0 < \omega_i < 1$ for $1 \leq i \leq n$ and for every intermediary node i, the probability that ω_i increases is equal to the probability that ω_i decreases.*

Given this, we define the set of limit stationary equilibria as follow.

DEFINITION 3. *Given a bargaining game Γ and a class of (k, T_k) replicated economies, a pair of state and strategy (ω, σ) is a* **limit stationary equilibrium** *of Γ, if it is balanced and for every $k \in \mathbb{N}^+$ there exist a semi-stationary equilibrium $(\omega(k), \sigma(k))$ in $\Gamma^k_{T_k}$ such that $\lim_{k\to\infty}(\omega(k), \sigma(k)) = (\omega, \sigma)$.*

Nonexistence of Limit Stationary equilibrium

In [2], it is shown that even when trade is beneficial and the efficient trading dynamic converges to a single and unique state, this dynamic cannot be consistent with the incentive structure of the bargaining process. Furthermore, it is shown that no converging trading dynamic can be consistent with the bargaining process. More specifically, [2] shows that:

> *If $V < (n+1)C_0 + nC_1 + \cdots + C_n$ then there does not exist any limit stationary equilibrium for $\delta > 1 - \epsilon$ with ϵ small enough.*

The main reason for such a result is the combination of the network structure and the given bargaining process. In this model, when selected, an agent can find a potential trading partner immediately. That is, there is no search friction among agent in neighboring nodes. In the next section we will consider a variation of this model to capture search friction. The probability that an agent finds a potential trading partner depends on how many of them there are.

2.2 Bargaining with Search Friction

We adapt the same framework as described above, except in step 3 where we introduce search friction. As before we assume that the agents know the fraction of agents who hold an item at each neighboring location. Differing from the previous set-up, however, the agents do not know the identity of neighboring agents who possess the item and have to query them to determine this. We only allow agents to query one potential trading agent. If the selected agent i has the item, then she queries one of the agents in the next hop at random to find out if the queried agent would be willing to trade (at an appropriate price). Note that, in contrast to the early model, depending on the system state, there is a chance of querying an agent who already possesses the item, and therefore, is not interested in a trade with anyone from the previous hop. Similarly, when agent i does not have the item, she queries agents in the previous hop at random and can encounter an individual who also does not possess the item. If a query is not successful, then the game moves to the next period and restarts from Step 1. The definitions of the replicated economy, semi-stationary equilibria, balanced equilibria and limit stationary equilibria carry through unchanged.

Note that introducing search friction will result in some trades being delayed. The reason for this is that knowing the aggregate system state, the agent can anticipate the current likelihood of further trading the item, and so could be willing to accept the item only at an unacceptable price (to the proposer) if the likelihood is very low; the same trade could take place at a later time as the system state would have changed. This will result in a loss of efficiency. We will, therefore, also compare the above bargaining model with search friction with a system where the trading is efficient despite search friction. For ease of explanation, for the results we will start with the efficient trading dynamic and then discuss the bargaining with search friction dynamic.

3. RESULTS

We start by analyzing an efficient trading network, i.e., one where once the agent locates a feasible trading partner, then trade necessarily occurs. Since only the state of middlemen can change, the entire system can be represented by a vector-valued random process $\{(X_1(t), X_2(t), \ldots, X_n(t))\}_{t=1}^{\infty}$ where we keep track of the number of agents who have the item at each level. For mathematical convenience we will append $X_0(t) \equiv km$ and $X_{n+1}(t) \equiv 0$ for the states of the sellers and the buyers, respectively. With probability $\frac{X_i(t)}{kmN}\left(1 - \frac{X_{i+1}(t)}{km}\right)$ agent i is selected, has an item and locates a neighboring agent that needs the item, and with probability $\frac{km-X_i(t)}{kmN}\frac{X_{i-1}(t)}{km}$ agent i is selected, needs the item and locates a neighboring agent that has the item. This shows that we have a Markov process. Since the (scaled) transition matrix satisfies Lipschitz conditions, we analyze the fluid limit that is obtained by scaling time and space, which is the same as the limit of replication. By an application of Kurtz's Theorem [1, Th. 2.1, Chapter 11], we obtain a differential equation for the limiting process that lives in a compact space. Then, using a fixed point argument, we can show that the resulting dynamics have a unique equilibrium that is globally asymptotically stable. For each k, it is easy to see that the Markov process is irreducible and has finite states, and so is positive recurrent. Thus, owing to the compact setting, the stationary measures of the scaled state processes converge to the point mass on the equilibrium point as k increases without bound. We summarize this in the following theorem.

THEOREM 1. *With search friction, if trade is efficient (no delay), then the trading dynamic converges to an unique state.*

For $n = 1$ we can explicitly characterize the limiting equilibrium state x_1^*. We will describe the equilibrium state for a generalization of the model described earlier. Let a be the (asymptotic) fraction of sellers with the good and b the (asymptotic) fraction of buyers who need the good. In addition, let each type of agent be chosen using a distribution $\{p_0, p_1, p_2\}$ such that $p_0+p_1 > 0$ and $p_1+p_2 > 0$, within each location agents are still chosen at random. Then $x_1^* = \frac{a(p_0+p_1)}{a(p_0+p_1)+b(p_1+p_2)}$, which for our model setting, i.e., $a = 1$, $b = 1$, $p_i \equiv 1/(n+1)$, yields $x_1^* = 1/2$.

Next we present a result that further characterizes the efficient equilibrium state and compares that to a potential bargaining equilibrium.

THEOREM 2. *The equilibrium of efficient trading dynamics is not a limit stationary equilibrium of a bargaining game for some range of parameters. There exists $a_0 > 1$; $a_1, \ldots, a_n \geq 1$ such that if $V < \sum_i a_i C_i$, then with δ close to 1, the efficient (no delay) trading process cannot be the limit equilibrium of the bargaining game.*

As an example, consider $n = 1$. As discussed above the efficient trading dynamic leads to the unique stationary state $x_1^* = 1/2$. However, in order for the bargaining process be consistent with this dynamic, the limit payoff need to satisfy certain conditions. See [2] for a reasoning of how these conditions are derived. In this example, these conditions requires $V > \frac{3}{2}C_0 + C_1$. Thus, if $V < \frac{3}{2}C_0 + C_1$ and the discount rate δ approaches 1, the efficient (no delay) trading process cannot be the limit equilibrium of the bargaining game.

However, unlike [2], where it is proved that in fact the bargaining process cannot be consistent with any converging trading process, here we show that there exists a limit stationary equilibrium. Due to the result above, this process is not efficient. For example, consider the case $n = 1$ and $V < \frac{3}{2}C_0 + C_1$. The no delay trading process cannot be the limit equilibrium of the bargaining game, but one can construct a limit stationary equilibrium, where agents only trade with a probability $p < 1$ along the first link, while they always trade along the second. As a result, in the limit only a small fraction of middlemen hold the good: $x_1^* < 1/2$. This will lower the bargaining power of buyers (agents at node 2) because it becomes harder for them to find a middlemen with an available item. This effect will change some of the conditions that could have been derived if agents always trade whenever they can. More formally, we have the following.

THEOREM 3. *In the bargaining game with search friction, there always exists a limit stationary equilibrium.*

4. REFERENCES

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